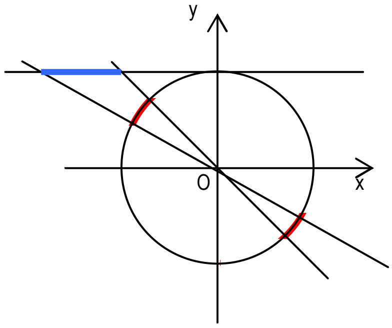
16. $ctg^2 x + (1+\sqrt{3})\, ctg\, x + \sqrt{3} < 0$

$$ctg\, x = \frac{-1-\sqrt{3} \pm \sqrt{1+2\sqrt{3}+3-4\sqrt{3}}}{2} = \frac{-1-\sqrt{3} \pm (\sqrt{3}-1)}{2} = \left\langle \begin{array}{l} -\sqrt{3} \\ -1 \end{array} \right.$$

$$-\sqrt{3} < ctg\, x < -1$$



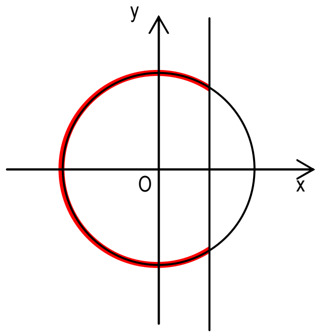
$$\frac{3}{4}\pi + k\pi < x < \frac{5}{6}\pi + k\pi$$

17. $2\, sen^2 x + 4\cos^2 x > 5\cos x$

$2(1-\cos^2 x) + 4\cos^2 x - 5\cos x > 0 \qquad 2\cos^2 x - 5\cos x + 2 > 0$

$$\cos x = \frac{5 \pm \sqrt{25-16}}{4} = \left\langle \begin{array}{l} 2 \\ \frac{1}{2} \end{array} \right. \qquad \cos x < \frac{1}{2} \quad \vee \quad \cos x > 2$$

$$\Rightarrow \qquad \cos x < \frac{1}{2}$$



$$\frac{\pi}{3} + 2k\pi < x < \frac{5}{3}\pi + 2k\pi$$

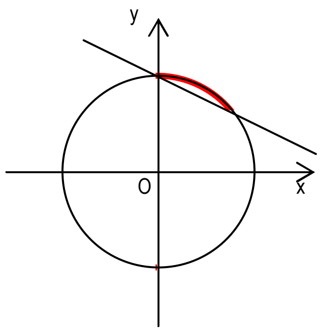
18. $\cos x + \sqrt{3}\, sen\, x - \sqrt{3} > 0$

Si tratta di una disequazione lineare, che può essere risolta graficamente: $sen\, x = Y \qquad \cos x = X$

$$\begin{cases} X + Y\sqrt{3} - \sqrt{3} = 0 \\ X^2 + Y^2 = 1 \end{cases} \qquad \begin{cases} X = -Y\sqrt{3} + \sqrt{3} \\ 3Y^2 - 6Y + 3 + Y^2 = 1 \end{cases} \qquad \begin{cases} X = -Y\sqrt{3} + \sqrt{3} \\ Y = \dfrac{3 \pm \sqrt{9-8}}{4} = \left\langle \begin{array}{l} 1 \\ \frac{1}{2} \end{array} \right. \end{cases}$$

$$\begin{cases} X = 0 \\ Y = 1 \end{cases} \qquad x = \frac{\pi}{2} + 2k\pi$$

$$\begin{cases} X = \dfrac{\sqrt{3}}{2} \\ Y = \dfrac{1}{2} \end{cases} \qquad x = \frac{\pi}{6} + 2k\pi$$



$$\frac{\pi}{6} + 2k\pi < x < \frac{\pi}{2} + 2k\pi$$

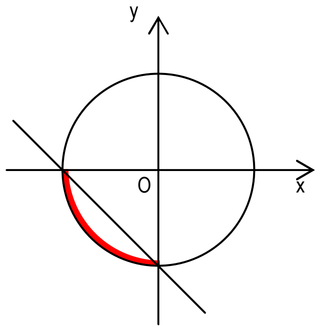
19. $sen\,x + \cos x + 1 < 0$

Si tratta di una disequazione lineare, che può essere risolta graficamente: $sen\,x = Y \qquad \cos x = X$

$$\begin{cases} Y + X + 1 = 0 \\ X^2 + Y^2 = 1 \end{cases} \qquad \begin{cases} X = -Y - 1 \\ Y^2 + 2Y + 1 + Y^2 = 1 \end{cases} \qquad \begin{cases} X = -Y - 1 \\ 2Y^2 + 2Y = 0 \end{cases}$$

$$\begin{cases} X = -1 \\ Y = 0 \end{cases} \qquad x = \pi + 2k\pi$$

$$\begin{cases} X = 0 \\ Y = -1 \end{cases} \qquad x = \frac{3}{2}\pi + 2k\pi$$



$$\pi + 2k\pi < x < \frac{3}{2}\pi + 2k\pi$$

20. $\cos x + (\sqrt{2} - 1)\, sen\,x + 1 < 0$

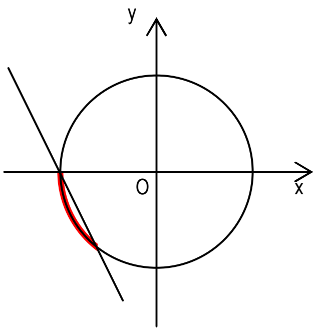
Si tratta di una disequazione lineare, che può essere risolta graficamente: $sen\,x = Y \qquad \cos x = X$

$$\begin{cases} X + Y(\sqrt{2} - 1) + 1 = 0 \\ X^2 + Y^2 = 1 \end{cases} \qquad \begin{cases} X = -Y(\sqrt{2} - 1) - 1 \\ (3 - 2\sqrt{2})\,Y^2 + 2Y(\sqrt{2} - 1) + 1 + Y^2 = 1 \end{cases}$$

$$\begin{cases} X = -Y(\sqrt{2} - 1) - 1 \\ 2\sqrt{2}(\sqrt{2} - 1)\,Y^2 + 2Y(\sqrt{2} - 1) = 0 \end{cases} \qquad \begin{cases} X = -Y(\sqrt{2} - 1) - 1 \\ \sqrt{2}\,Y^2 + Y = 0 \end{cases}$$

$$\begin{cases} X = -1 \\ Y = 0 \end{cases} \qquad x = \pi + 2k\pi$$

$$\begin{cases} X = -\dfrac{\sqrt{2}}{2} \\ Y = -\dfrac{\sqrt{2}}{2} \end{cases} \qquad x = \frac{5}{4}\pi + 2k\pi$$



$$\pi + 2k\pi < x < \frac{5}{4}\pi + 2k\pi$$

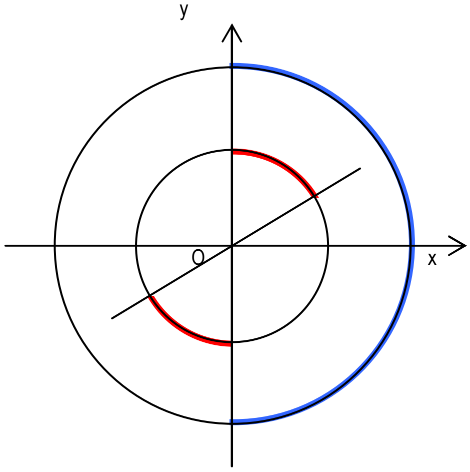
21. $\sqrt{3}\, sen\, 2x - \cos 2x \geq 0$

Trattandosi di una disequazione omogenea di primo grado, si procede raccogliendo il termine cos 2x:

$$\cos 2x\,(\sqrt{3}\, tg\, 2x - 1) \geq 0$$

$$\cos 2x \geq 0$$

$$tg\, 2x \geq \frac{\sqrt{3}}{3}$$



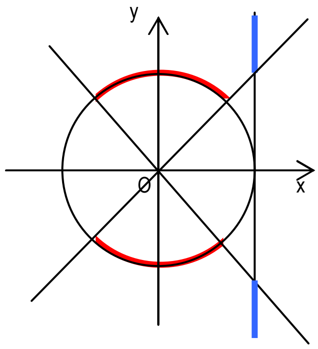
$$\frac{\pi}{6} + 2k\pi \leq 2x \leq \frac{7}{6}\pi + 2k\pi$$

$$\frac{\pi}{12} + k\pi \leq x \leq \frac{7}{12}\pi + k\pi$$

22. $sen^2 x - \cos^2 x > 0$

Disequazione omogenea di secondo grado. Dividiamo entrambi i membri per $\cos^2 x$:

$$tg^2 x - 1 > 0 \quad \Rightarrow \quad tg\, x < -1 \ \vee\ tg\, x > 1$$



$$\frac{\pi}{4} + k\pi < x < \frac{3}{4}\pi + k\pi$$

Soluzione alternativa:

$$-(-sen^2 x + \cos^2 x) > 0$$

$$\cos^2 x - sen^2 x < 0$$

$$\cos 2x < 0 \quad \Rightarrow \quad \frac{\pi}{2} + 2k\pi < 2x < \frac{3}{2}\pi + 2k\pi \quad \Rightarrow \quad \frac{\pi}{4} + k\pi < x < \frac{3}{4}\pi + k\pi$$

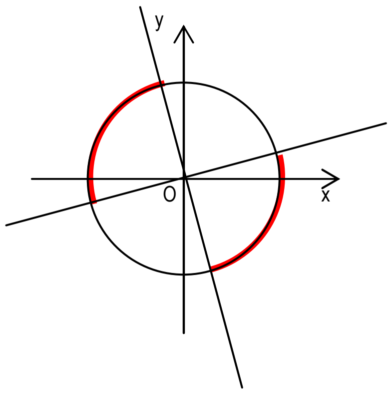
23. $\cos^2 x - 2\sqrt{3}\cos x\, sen\, x - sen^2 x > 0$

Disequazione omogenea di secondo grado. Dividiamo entrambi i membri per $\cos^2 x$:

$$tg^2 x - 2\sqrt{3}\, tg\, x - 1 < 0$$

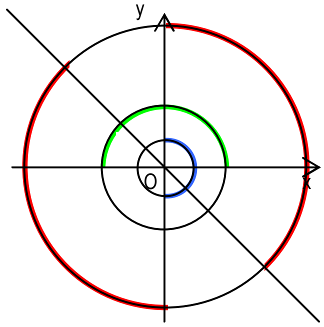
$$tg\, x = -\sqrt{3} \pm \sqrt{3+1} = -\sqrt{3} \pm 2$$

$$-\sqrt{3} - 2 < tg\, x < -\sqrt{3} + 2$$



$$-\frac{5}{12}\pi + k\pi \leq x \leq \frac{\pi}{12} + k\pi$$

24. $sen^2 x + sen\,x \cos x < 0$



$sen\,x \cos x\,(tg\,x + 1) < 0$

$sen\,x > 0$

$\cos x > 0$

$tg\,x > -1$

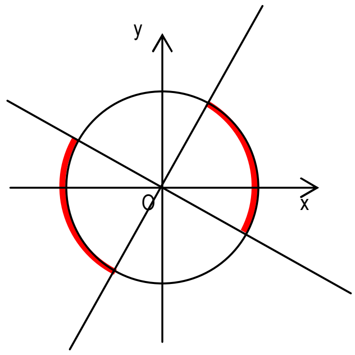
$$\frac{3}{4}\pi + k\pi < x < \pi + k\pi$$

25. $6\,sen^2 x - 2\sqrt{3}\,sen\,x \cos x < 3$

Disequazione riconducibile a omogenea di secondo grado: $6\,sen^2 x - 2\sqrt{3}\,sen\,x\cos x < 3\,(sen^2 x + \cos^2 x)$

$3\,sen^2 x - 2\sqrt{3}\,sen\,x\cos x - 3\cos^2 x < 0$

Divido per $\cos^2 x$: $3\,tg^2 x - 2\sqrt{3}\,tg\,x - 3 < 0$



$$tg\,x = \frac{\sqrt{3} \pm \sqrt{3+9}}{3} = \left\langle \begin{array}{l} \sqrt{3} \\ -\frac{\sqrt{3}}{3} \end{array} \right.$$

$$-\frac{\sqrt{3}}{3} < tg\,x < \sqrt{3}$$

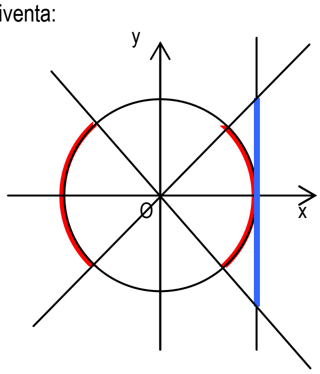
$$-\frac{\pi}{6} + k\pi < x < \frac{\pi}{3} + k\pi$$

26. $sen^4 x - \cos^4 x < 0$

$(sen^2 x - \cos^2 x)\,(sen^2 x + \cos^2 x) < 0$ che, per la prima relazione fondamentale, diventa:

$sen^2 x - \cos^2 x < 0$ disequazione omogenea di secondo grado, perciò divido per $\cos^2 x$:

$tg^2 x - 1 < 0 \quad \Rightarrow \quad -1 < tg\,x < 1$



$$-\frac{\pi}{4} + k\pi < x < \frac{\pi}{4} + k\pi$$